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Macroscopic estimation method of the mixedness of Kenics type static mixer

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Abstract−We investigated the mixing process of the Kenics type static mixer, in which the mixing is enhanced by both advective mixing and mixing due to molecular diffusion in order to propose an estimation method of the mixedness. The results show that the element divides, folds and stretches the mixing fluids and forms a lamellar structure with striation width l_a. The value of l_a decreases with an increase in the number of elements by l_a-$(2^{n-1})^{-1}$, which represents the characteristics length of advective mixing. The characteristic length of mixing due to molecular diffusion l_d can be estimated by solving the one-dimensional unsteady species conservation equation analytically. The macroscopic estimation and prediction method of mixedness is proposed as function of l_a and l_d. When $l_a > l_d$, the advective mixing is dominant, while, when $l_a < l_d$, the mixing due to molecular diffusion plays an important role.

Key words: Mixedness, Advective Mixing, Molecular Diffusion, Kenics Static Mixer

INTRODUCTION

Mixing technology plays an important role in many engineering fields, such as chemical engineering, mechanical engineering, bio engineering and so forth.

Static mixers are widely used because continuous mixing can be performed with no moving parts [1].

A Kenics static mixer is a typical static mixer that is used in many applications, andthe mixing mechanism has been studied both experimentally and numerically.

Ottino [2] emphasized that the mixing mechanism consists of fluid mechanics plus diffusion plus reaction. And these factors have to be simultaneously considered and superimposed for precise analysis.

Ottino [3] studied the mixing with diffusion and reaction in terms of the concept of deformation of material surfaces. He points out that the lamellar structure, which enlarges the surface between two fluids of mixing by deformation of the material surface, is essential for advective mixing.

Particle tracking simulations were used to compute residence time distributions, striation evolution, and variation coefficient as a function of the number of elements. The results calculated from the numerical simulation agree closely with the experimental results. Stretching of material elements in the mixer flow was also computed. The average stretching of material elements increased exponentially with the number of periodic mixer segments, which shows the signature of chaotic flows [4].

Hobbs and Muzzio [5] studied numerically the effects of injection location, flow ratio and geometry on Kenics mixer performance. In this study, CFD methods were used to analyze the numerical analysis. The results revealed that the injection location is unimportant if a sufficiently long mixer is available. On the other hand, the location of the initial injection has a large effect on the spread of tracer over the first few mixer elements. For typical industrial application using a 6- or 12-element Kenics static mixer, this dependence on inlet conditions could play a significant role in mixing quality [5]. Although they estimated the statistical quantities of mixedness such as mixture variance, no quantitative estimation of mixedness is proposed, taking into account both due to the advective mixing and due to molecular diffusion.

Hobbs and Muzzio [6] investigated the Reynolds number effect on laminar mixing in a Kenics static mixer. They indicated that effective mixing was achieved for creeping flow condition (Re≤10) [6].

The flow in the Kenics static mixer in the range of Re=100-1,000 was investigated in detail both numerically and experimentally. It was found that at Re=300 the flow becomes unsteady. Two numerical methods, the lattice Boltzmann (LB) method and FLUENT, were used to simulate the flow. Furthermore, the flow field and dynamic behavior were validated by means of LDA experiments [7].

These studies found that mixing due to chaotic advection, that is an advective mixing, which enhances the mixing by spreading striations due to stretching and folding, plays an important role in addition to the molecular diffusion which makes the concentration uniform. In other words, both the lamellar structure by advective mixing and concentration uniformity due to molecular diffusion should be taken into account to estimate the mixedness of static mixers.

From a practical point of view, however, it is still difficult to predict the mixedness as a function of the number of elements, although many detailed studies have been performed. Thus, from the macroscopic point of view, a simple and reasonable method to predict the mixedness, taking into account both lamellar structure and concentration uniformity, is needed.

The present study, then, investigates the mixing process and evaluates the role of the advective mixing and the mixing due to molecular diffusion. The mixing pattern is numerically investigated, and the reliability of the numerical results is confirmed by the corresponding

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Fig. 1. Analytical model.

experimental results. And a procedure to predict the magnitude of mixing, that is the mixedness, in the Kenics static mixer from the macroscopic point of view is proposed.

NUMERICAL ANALYSIS

A flow in the Kenics static mixer with 10 elements as shown in Fig. 1 is numerically solved in a steady state condition. Here, the inner diameter of the pipe D is 20 mm and the length of elements L is 30 mm.

The governing equations are as follows:

The law of conservation of mass $(\nabla \cdot u)=0$ (1)

The law of conservation of momentum

$$u \cdot \nabla u = -\frac{1}{\rho}\nabla p + \nu \nabla^2 u \quad (2)$$

The fluid is assumed to be Newtonian incompressible viscous fluid as mentioned in detail later, "Flow conditions".

Fluent 6.2 (Ansys Co.) was used for numerical calculation. A mesh using the calculation was drawn up by using Gambit (Ansys Co.). The mesh of one element is shown in Fig. 2. Hexahedral grids were used to apply Quadratic Upwind Interpolation for Convective Kinematics (QUICK) in order to reduce numerical error [8]. About 1,120,000 grids were used.

The mixing pattern is visualized by setting a particle in one side of the inlet flow. Thus, it shows the pattern due to the advective mixing and no influence of molecular diffusion is taken into account.

EXPERIMENTAL ANALYSIS

Fig. 3 shows the experimental apparatus used in the present study.

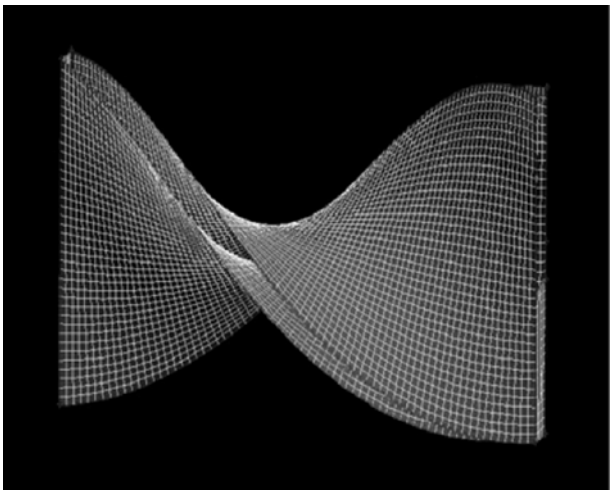

Fig. 2. Analytical mesh of one element.

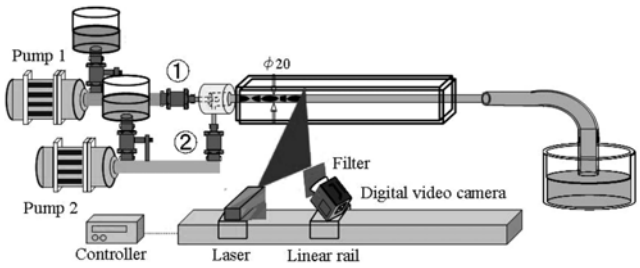

Fig. 3. Experimental apparatus.

The laser induced fluorescence (LIF) method was used to visualize cross-sectional images of mixing.

The experimental setup consists of a fluid supply section, mixing section and visualization unit. The fluid supply section has two tanks to store the working fluid. Rhodamine B was mixed in one tank for visualization.

The clear Kenics static mixer is used to be able to visualize the mixing motion. One to six clear element units were used as shown in Fig. 4(a). The twist angle of a mixing element is 180° and two consecutive elements alternate in rotation direction. The clear element unit was inserted into the clear pipe to form one Kenics static mixer shown in Fig. 4(b).

A rectangular box that was filled with working fluid is set around the mixing section to reduce the lensing effects caused by the pipe wall curvature as shown in Fig. 5. A CCD camera and laser source is set at the visualization unit. The light source was a diode laser with a wavelength of 532 nm and a lens group is used to create a laser light sheet.

The laser light sheet was set just downstream of the element unit

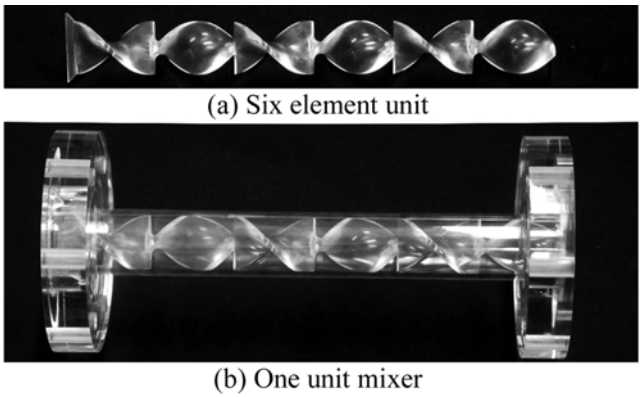

Fig. 4. Clear Kenics static mixer.

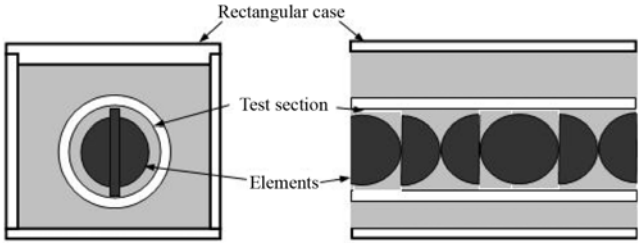

Fig. 5. Schematic of mixing parts.

Fig. 6. Inlet fluid configuration.

to visualize the mixing after elements. When the fluid with Rhodamine B passed the laser sheet, the fluid pattern with Rhodamine B was visualized in the cross-section of the laser sheet. The visualized image was recorded by a CCD camera. A band pass filter (MELLES GRIOT, 03FCG089) was set in front of the camera lens to observe only the image of the fluorescence of Rhodamine B.

FLOW CONDITIONS

In the present study, the mixing process in the Kenics static mixer is investigated. The inlet fluid conditions are systematically varied. Two inlet fluid configurations are investigated; one is the line-symmetric configuration as shown in Fig. 6(a) and the other is the axi-symmetric configuration as shown in Fig. 6(b).

In the case of the line-symmetric condition, the region ratio of two fluids, that is, the ratio of the fluid of black region and that of white region at the inlet, is set as unity. In the case of the axi-symmetric condition, the region ratios 1, 1/2, 1/4, and 1/8 are investigated. The inlet fluid conditions are, then, summarized as shown in Table 1.

The working fluid in experiments was a 98% glycerin aqueous solution, which was assumed to be Newtonian fluid. The property of the working fluid is obtained as a mixture of glycerin and water by using the property values of each fluid in Table 2.

The mean inlet velocity is set as 2.49 mm/s and the Reynolds number, based on the mean inlet velocity and the pipe diameter, is estimated as 0.06 to keep the Reynolds number much less then 300 [7].

The mixing process for all five cases has been numerically investigated. Experimental investigations for two limiting cases, Case 1 and Case 5, have been performed to confirm the numerical results.

Table 1. Inlet fluid conditions

	Configuration	Region ratio
Case 1	line-symmetric	1
Case 2	axi-symmetric	1
Case 3	axi-symmetric	1/2
Case 4	axi-symmetric	1/4
Case 5	axi-symmetric	1/8

Table 2. Properties of Working fluids (20 °C)

	Density ρ (kg/m^3)	Viscosity (Pa·s)
Glycerin $C_3H_5(OH)_3$	1254	0.974
Water	998.2	0.001

In the numerical calculation, a uniform velocity profile at the inlet is assumed and free outflow is assumed at the outlet.

RESULTS AND DISCUSSION

1. Mixing Pattern

Fig. 7 shows the numerical and experimental results of cross sectional images of the mixing pattern. The LIF images coincide well with the dark region of the numerical results, for both Case 1 and Case 5. This indicates that the numerical results in the present study show mixing patterns with good accuracy. The results show that the fluid blob (black region of numerical result) is subject to stretching and folding, forming striation and the striation spread over the cross section.

In experiments, the image after six elements becomes slightly ambiguous compared to the numerical results. This is due to the molecular diffusion, while no molecular diffusion is taken into account in the numerical results.

In the line-symmetric case (Case 1), two fluids, the fluid in the black region and that in the white region, are stretched by a factor of two and divided by two for one element.

In the Case 2, the stretching of fluid is less than that of Case 1 notwithstanding the region ratio of two fluids at the inlet region is the same, which is unity. It shows that the inlet pattern of two fluids is important to the mixedness. When the region ratio becomes far from unity, the mixing pattern becomes sparse. It indicates many elements are necessary to obtain the same level of mixedness. These results indicate that the initial pattern of the two fluids and the region ratio of two fluids at the inlet are important for the mixedness of the fluids.

2. Estimation of the Mixedness

2-1. Lamellar Structure

As pointed out by Ottino [2], the lamellar structure is essential and the deformation of the fluid blob, that is, the advective mixing, and molecular diffusion have to be analyzed simultaneously to analyze the mixing process precisely. From a practical viewpoint, however, it would be useful if the mixedness could be predicted by a reasonable estimation method. Thus, in the present study, the estimation using characteristic width is proposed by introducing a characteristic width of advective mixing (l_a) and that of mixing due to molecular diffusion (l_d).

Fig. 8 shows a schematic of the local structure of the lamellar pattern along the coordinate normal to the iso-concentration line. In Fig. 8, l_a is the characteristic width of advective mixing and l_d is that of the mixing due to molecular diffusion. Details of l_a and l_d are given in Secs 2.2 and 2.3, respectively.

ΔC is the concentration difference, which indicates a magnitude of the uniformity of the concentration. When $\Delta C=0$, the concentration is completely uniform.

When $l_a > l_d$, the advective mixing is dominant and then the fineness of the lamellar structure shows the mixedness. On the other hand, when $l_d > l_a$, the molecular diffusion is dominant and then the uniformity of the concentration shows the mixedness.

As shown in Fig. 8, l_a is estimated as a lamellar width, and l_d can be estimated as a width of one-dimensional molecular diffusion.

2-2. Characteristic Width of Advective Mixing

Ottino [9] reported that the mixing in the Kenics static mixer is approximated by Baker's transformation, and then the striation is

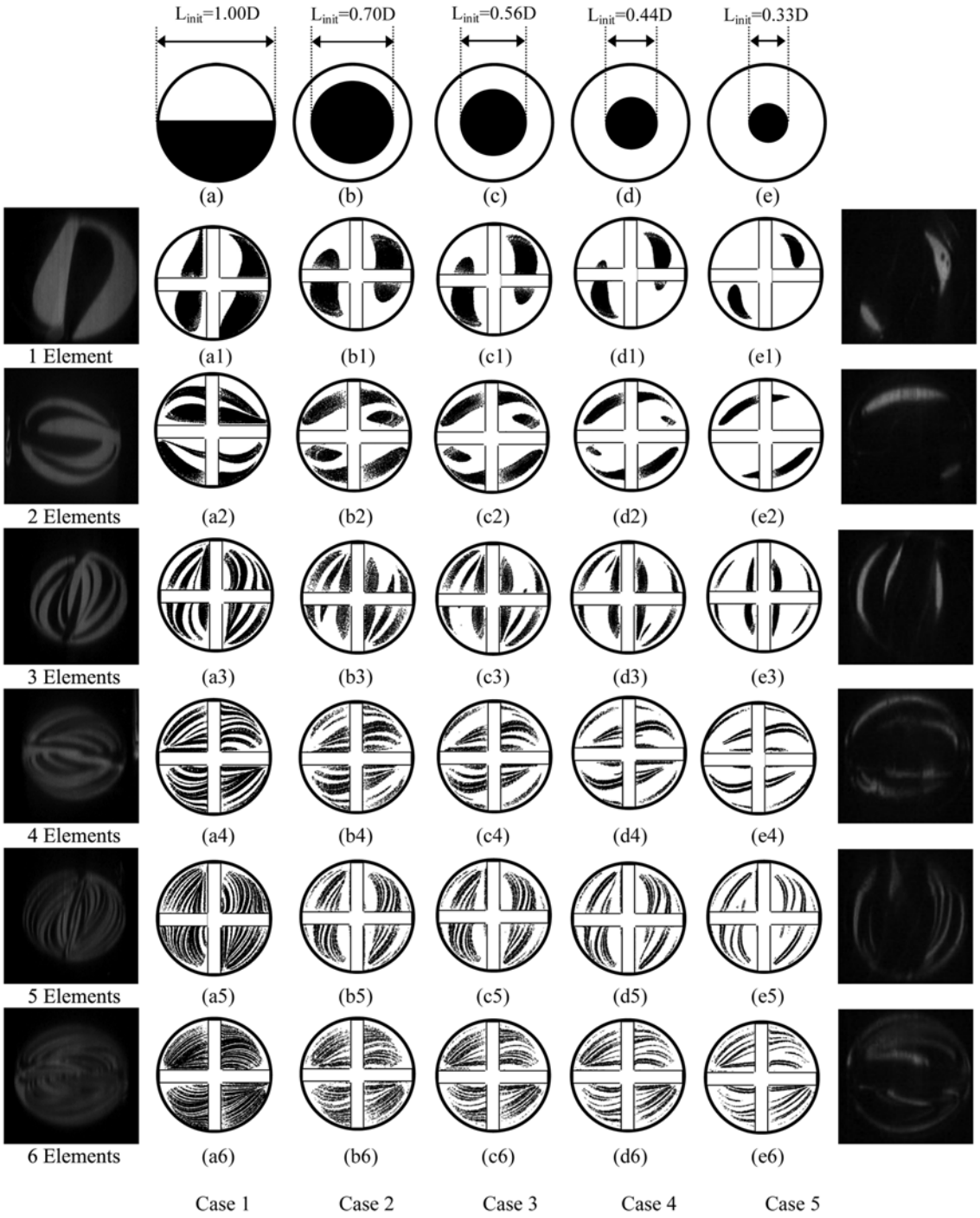

Fig. 7. Experimental and numerical results of the cross-sectional mixing pattern.

stretched twice and folded by each element, forming a lamellar structure, and consequently the striation length becomes 2^n and the striation width, which is equivalent to the lamellar width, becomes 2^{-n}, where n is the number of elements.

This rule is strictly applied for the line-symmetric case because the initial characteristic length is unity as shown in Fig. 7(a). As for the axi-symmetric configuration, the rule can be well applied if the appropriate initial characteristic length is determined.

When the initial characteristic length is determined as the diameter of the center fluid blob (black area), the variation in the striation length as a function of the elements is estimated as shown in Table 3. The variation in Table 3 fits well with that in the length of the striation in Fig. 7. These results indicate that the striation length is estimated as

$$l_s = L_{init} \cdot 2^{n-1} \tag{3}$$

and then the striation width is

$$w_s = \frac{1}{L_{init} \cdot 2^{n-1}} \tag{4}$$

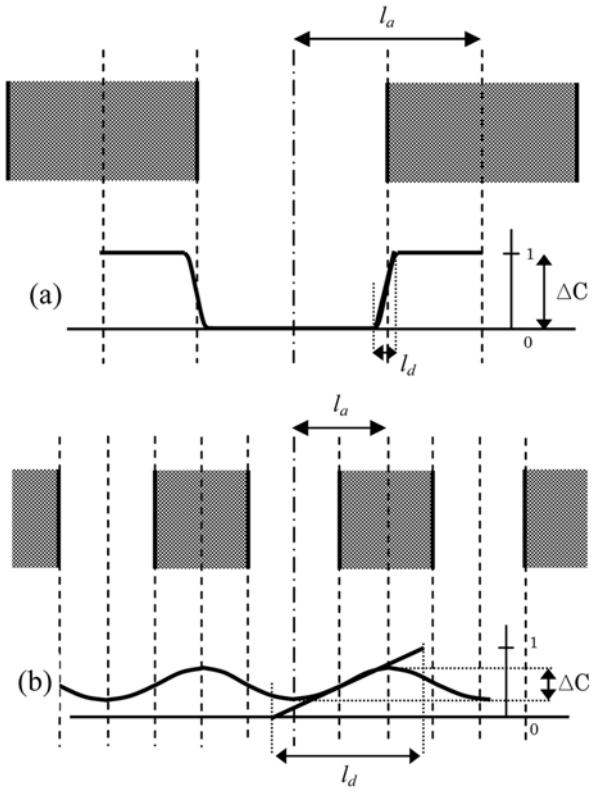

Fig. 8. Schematic of lamellar structure.

Table 3. Estimated Striation length (L_s/D)

Inlet parts \ Elements	L_{init}	1	2	3	4	5
Case 1	1	1	2	4	8	16
Case 2	0.70	0.7	1.4	2.8	5.6	11.2
Case 3	0.56	0.56	1.12	2.24	4.48	8.96
Case 4	0.44	0.44	0.88	1.76	3.52	7.04
Case 5	0.33	0.33	0.66	1.32	2.64	5.28

Since the stretching and folding occur due to the advection, the striation width is considered to be the characteristic width of advective mixing. Consequently, l_a is estimated by the equation below:

$$l_a = w_s = \frac{1}{L_{init} \cdot 2^{n-1}} \tag{5}$$

2-3. Characteristic Width of Mixing Due to Molecular Diffusion

The characteristic width of mixing due to molecular diffusion can be estimated by a one-dimensional concentration profile normal to the iso-concentration line. Thus, the one-dimensional unsteady differential equation of molecular diffusion is solved analytically as follows:

$$c_t = \alpha^2 c_{xx},\ 0<t<\infty,\ 0\le x\le 1 \tag{6}$$

Boundary condition: $c_x(0, t)=0,\ 0<t<\infty$

$c_x(1, t)=0,\ 0<t<\infty$

Initial condition: $c(x, 0)=\varphi(x),\ 0\le x\le 1$

$\varphi(x)=0,\ 0\le x\le 0.5$

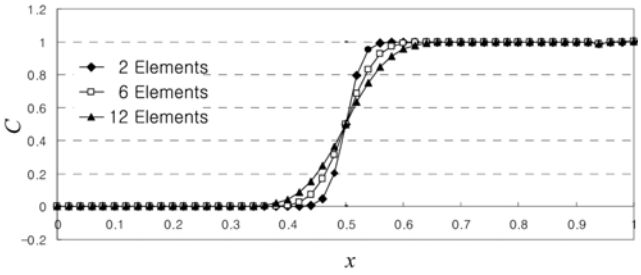

Fig. 9. Variation in the concentration variation of molecular diffusion.

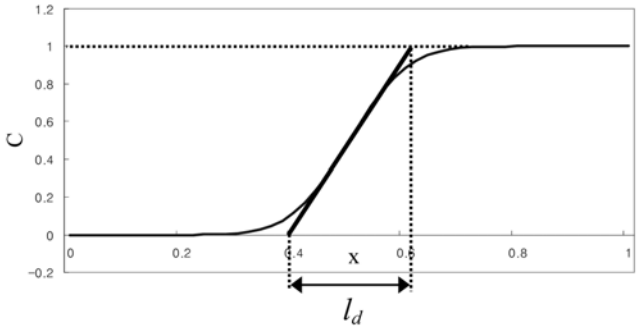

Fig. 10. Definition of l_d.

$\varphi(x)=1,\ 0.5\le x\le 1$

Here, x: coordinate [m]

t: time [s]

α: diffusion coefficient [m^2/s]

The solution is as follows:

$$c(x,t) = 0.5 - \sum_{n=1}^{\infty} A_n e^{-(\alpha n\pi)^2 t} \cdot \cos m\pi x \tag{7}$$

$$A_m = \frac{2}{m\pi}\sin(0.5m\pi)$$

The boundary conditions at both ends are set as zero concentration gradient condition, which is the Neuman condition. The initial condition is set as a step function from 0 to 1 at x=0.5. The result is shown in Fig. 9, assuming $\bar{u}$ =2.49 mm/s.

The characteristic width of molecular diffusion (l_d) is defined as shown in Fig. 10, using the analytical solution of the concentration profile of Eq. (7).

l_d is defined as the width between the cross point at C=0 and C=1 of the tangential straight line of the concentration profile of Eq. (7) at C=0.5 as shown in Fig. 10.

2-4. Estimation of the Magnitude of the Uniformity

To quantify the magnitude of the uniformity, the concentration variation ΔC is introduced. ΔC is defined as the concentration variation within l_a.

Fig. 11 shows the schematic of ΔC. When $l_a > l_d$ (Fig. 11(a)), the concentration variation ΔC is unity because the molecular diffusion occurs within l_a. When $l_a < l_d$ (Fig. 11(b)), the molecular diffusion spreads over the area of l_a. In this case, since the concentration can vary only within l_a as shown in Fig. 11(b), ΔC can be estimated as a concentration variation within l_a as well as the case of

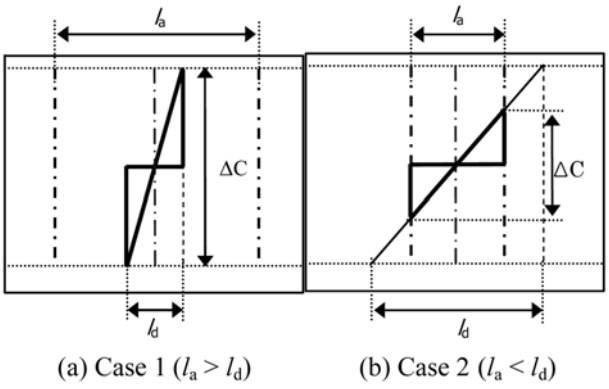

Fig. 11. Estimation method of concentration variation.

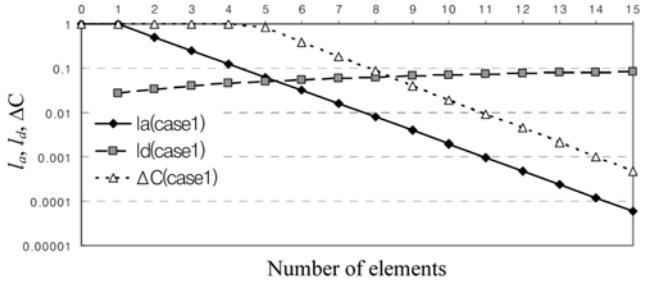

Fig. 12. Variations in the characteristic width of advective mixing and molecular diffusion, and the concentration variation.

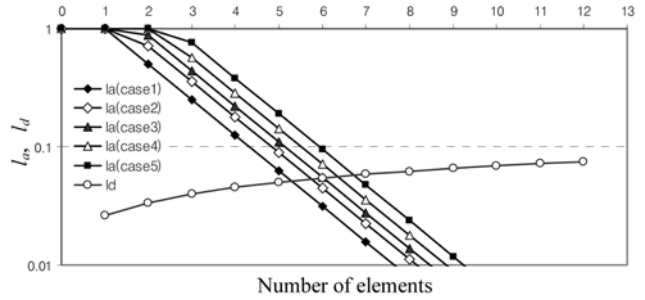

Fig. 13. Variation in the characteristic widths of advective mixing and molecular diffusion.

$l_a > l_d$. In this case, ΔC is less than unity.

2-5. Estimation Method of the Mixedness

Fig. 12 shows the variation in l_a, l_d and ΔC as a function of the number of elements. l_a decreases monotonically following Eq. (5), while l_d gradually increases with increasing the number of elements. The lamellar structure is dominant when $l_a > l_d$, indicating ΔC=1. Thus, the mixedness is characterized by the fineness of the lamellar structure.

The value of ΔC starts to decrease from the point where $l_a = l_d$ and the molecular diffusion gradually plays an important role. In other words, the concentration variation decreases toward the uniform concentration condition. In fact, ΔC becomes nearly 0.01 at the 10th elements. This suggests that fluids are supposed to mix almost uniformly. In this case, the mixedness is characterized by the uniformity of the concentration.

Figs. 13 and 14 show variation in l_a and l_d and that of ΔC as a function of the size of the initial fluid blob. These indicate that when the initial fluid blob becomes small, it needs more elements to reach similar mixing characteristics. These results indicate that macroscopic mixing characteristics can be predicted by using l_a and l_d. When the initial characteristic length L_{init} is determined, the variation in l_a is determined by Eq. (5). l_d can be determined by Eq. (7) when the fluids are determined. In addition, ΔC can be estimated by l_a and l_d as mentioned in Section 2.4.

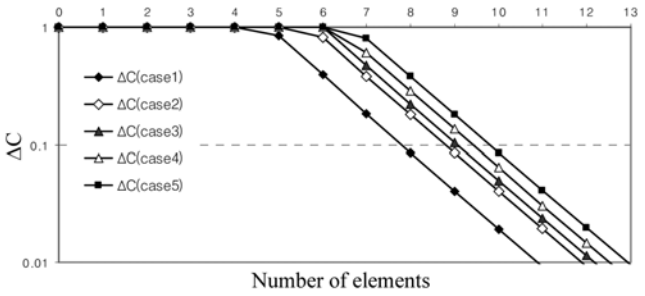

Fig. 14. Variation in the concentration.

CONCLUSION

We investigated the mixing process in the Kenics type static mixer in detail experimentally and numerically in order to propose a method of estimating the mixedness. Mixing due to advection and that due to molecular diffusion plays an important role in the mixing mechanism.

Based on experimental and numerical results, we proposed a macroscopic estimation and prediction method of the mixedness as a function of the number of elements. The characteristic width of the mixing due to advection l_a was estimated as a striation width, which is a function of the number of elements and the initial length of the fluid blob.

The characteristic width of mixing due to molecular diffusion l_d was estimated by solving the one-dimensional unsteady species conservation equation analytically. In addition, the concentration variation ΔC, which is the characteristic value of non-uniformity of concentration, was estimated by using l_a and l_d.

When $l_a > l_d$, the advective mixing is dominant and the mixing is characterized by the fineness of the lamellar structure of striation.

When $l_a < l_d$, the mixing due to molecular diffusion plays an important role and the mixing is characterized by the uniformity of the concentration.

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NOMENCLATURE

c : concentration [-]
D : diameter of mixing pipe [m]
l_a : characteristic width of advective mixing [m]
l_d : characteristic width of mixing due to molecular diffusion [m]

L_{init} : length of inlet pipe [m]
L_s : striation length [m]
n : number of element [-]
P : pressure [Pa]
u : velocity [m/s]
$\bar{u}$: mean velocity [m/s]
t : time [s]
w_s : striation width [m]
x : coordinate [m]
α : diffusion coefficient [m^2/s]
ρ : density [kg/m^3]
ν : viscosity [Pa·s]
φ : conditional expression [-]

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